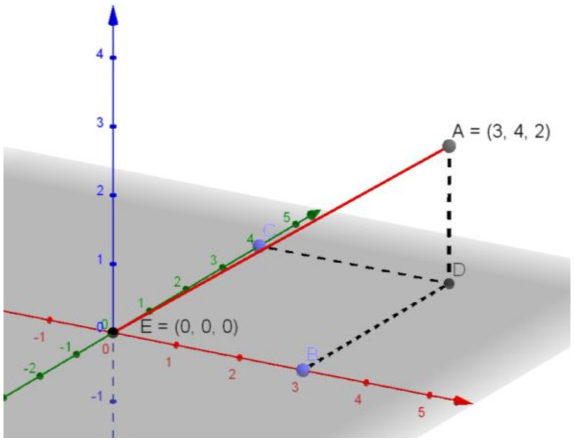
1) Calcular la distancia entre el punto P y el origen de coordenadas:

a) P = (3 ; 4; 2) **b) P = (-3 ; 4; 2)** **c) P = (-3 ; -2; 2)**

a) **P = (3 ; 4; 2)**

$$Dist_{OP} = \sqrt{x^2 + y^2 + z^2}$$

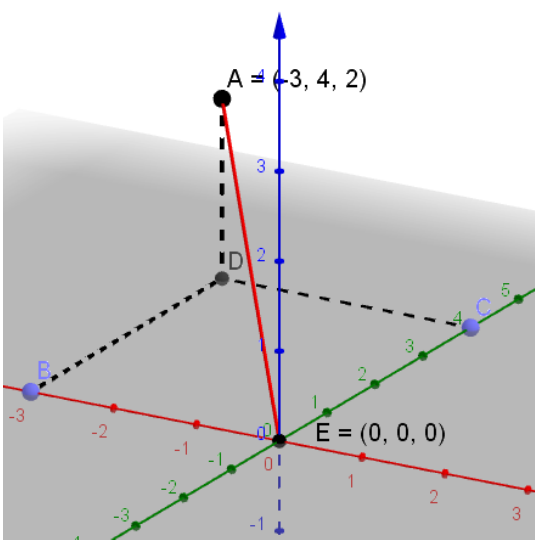
$\sqrt{3^2 + 4^2 + 2^2} = \sqrt{29}$ = **5,38**



b) **P = (-3 ; 4; 2)**

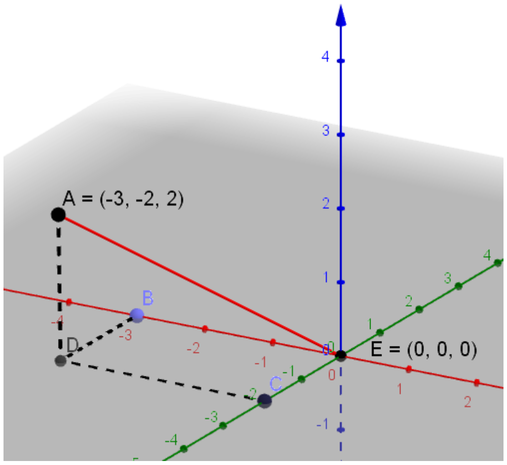
$$Dist_{OP} = \sqrt{x^2 + y^2 + z^2}$$

$\sqrt{(-3)^2 + 4^2 + 2^2} = \sqrt{29}$ = **5,38**



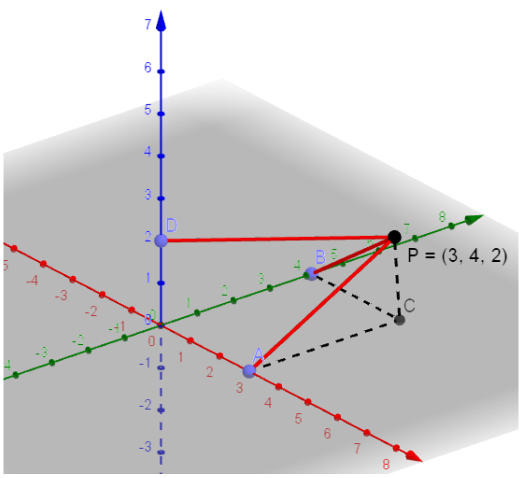
c) P = (-3 ; -2; 2))

$$Dist_{OP} = \sqrt{x^2 + y^2 + z^2}$$



$$\sqrt{(-3)^2 + (-2)^2 + 2^2} = \sqrt{17} = \mathbf{4{,}12}$$

2) Calcular las distancias desde el punto P = (3;4;2) hasta cada eje coordenado (x,y,z)



$$dist_{px} = \sqrt{(y)^2 + (z)^2} =$$

$$dist_{px} = \sqrt{(4)^2 + (2)^2} = \mathbf{4,47}$$

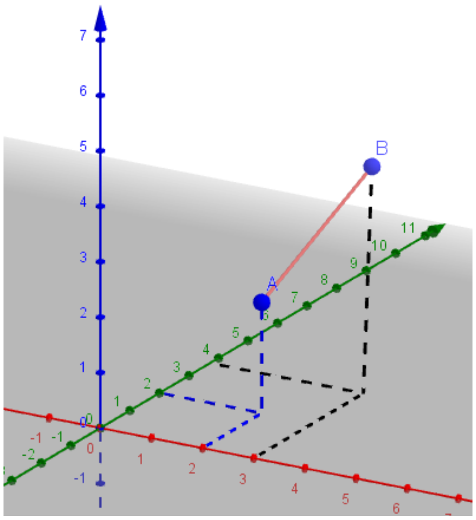
$$dist_{py} = \sqrt{(x)^2 + (z)^2} =$$

$$dist_{py} = \sqrt{(3)^2 + (2)^2} = \mathbf{3,60}$$

$$dist_{pz} = \sqrt{(y)^2 + (x)^2} =$$

$$dist_{pz} = \sqrt{(4)^2 + (3)^2} = \mathbf{5}$$

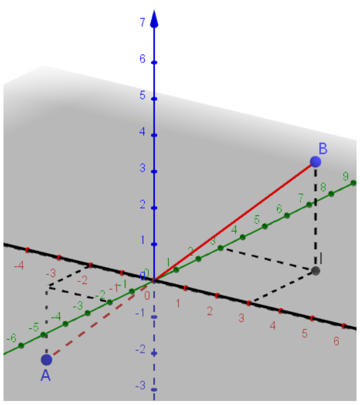
3) Calcular la distancia entre los puntos A = (2; 2; 2) y B = (3; 4; 4)



$$Dist_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_B)^2 + (z_B - z_A)^2}$$

$$Dist_{AB} = \sqrt{(3-2)^2 + (4-2)^2 + (4-2)^2} = \mathbf{3}$$

4) Calcular la distancia entre los puntos A = (-2; -2; -2) y B = (3; 3; 3)



$$Dist_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y)^2 + (z_B - z_A)^2}$$

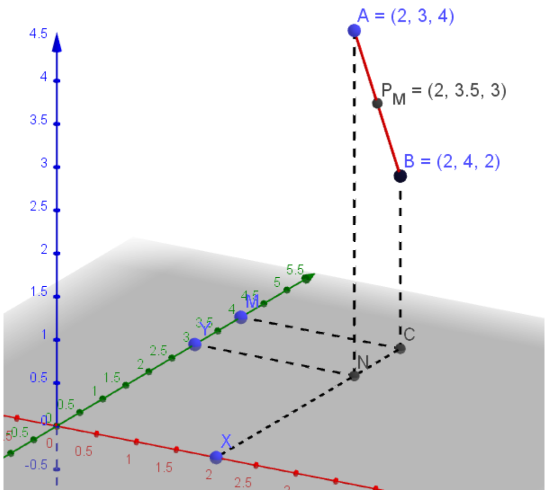
$$Dist_{AB} = \sqrt{(3-(-2))^2 + (3-(-2))^2 + (3-(-2))^2}$$

$$dist_{AB} = \sqrt{(5)^2 + (5)^2 + (5)^2} = \mathbf{8,66}$$

5) Calcular las coordenadas del punto medio del segmento $\overline{AB}$**:**

a) A = (2; 3; 4) B = (2; 4; 2) b) A = (-2; 3; -4) B = (2; 4; 2)

a) **A = (2; 3; 4) B = (2; 4; 2)**



$$X_M = \frac{X_A + X_B}{2} = \frac{2+2}{2} = 2$$

$$Y_M = \frac{y_A + y_B}{2} = \frac{3+4}{2} = 3,5$$

$$Z_M = \frac{z_A + z_B}{2} = \frac{4+2}{2} = 3$$

b) A = (-2; 3; -4) B = (2; 4; 2)

$$X_M = \frac{X_A + X_B}{2} = \frac{-2+2}{2} = 0$$

$$Y_M = \frac{y_A + y_B}{2} = \frac{3+4}{2} = 3,5$$

$$Z_M = \frac{z_A + z_B}{2} = \frac{-4+2}{2} = -1$$

6) Dado P = (2; 4; 6) calcular los cosenos directores y los ángulos de dirección.

$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{56}$ = 7,48

$\cos \propto = \frac{x}{\rho} = \frac{2}{\sqrt{56}} = \mathbf{0,267261241}$ => α = arc cos 0,267261241 = **74º 29´ 55,11”**

$\cos \beta = \frac{y}{\rho} = \frac{4}{\sqrt{56}} = \mathbf{0,534522483}$ => β = arc cos 0,534522483 = **57º 41´ 18,48”**

$\cos \gamma = \frac{z}{\rho} = \frac{6}{\sqrt{56}} = \mathbf{0,801783728}$ => γ = arc cos 0,801783728 = **36º 41´ 57,21”**

7) Calcular las coordenadas polares de los siguientes puntos:

a) A = (-3; 4; 1) b) B = (2; 2; 2)

a) A = (-3; 4; 1)

$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-3)^2 + 4^2 + 1^2} = \sqrt{26}$ = 5,10

$$\propto = \text{arc cos } \frac{x}{\rho} = \text{arc cos } \frac{(-3)}{\sqrt{26}} = \mathbf{126^{\text{º}}\ 1´\ 54,76"}$$

$$\beta = \text{arc cos } \frac{y}{\rho} = \text{arc cos } \frac{(4)}{\sqrt{26}} = \mathbf{38^{\text{º}}\ 20´\ 33,9"}$$

$$\gamma = \text{arc cos } \frac{z}{\rho} = \text{arc cos } \frac{(1)}{\sqrt{26}} = \mathbf{78^{\text{º}}\ 41´\ 32,17"}$$

b) B = (2; 2; 2)

$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(2)^2 + 2^2 + 2^2} = \sqrt{12}$ = 3,46

$$\propto = \text{arc cos } \frac{x}{\rho} = \text{arc cos } \frac{(2)}{\sqrt{12}} = \mathbf{54^{\text{º}}\ 41´\ 15,25"}$$

$$\beta = \text{arc cos } \frac{y}{\rho} = \text{arc cos } \frac{(2)}{\sqrt{12}} = \mathbf{54^{\text{º}}\ 41´\ 15,25"}$$

$$\gamma = \text{arc cos } \frac{z}{\rho} = \text{arc cos } \frac{(2)}{\sqrt{12}} = \mathbf{54^{\text{º}}\ 41´\ 15,25"}$$

8) Dadas las coordenadas cartesianas de los siguientes puntos hallar las coordenadas cilíndricas:

a) A = (2; 2; 2) **b) B = (3; 2; 1)**

a) A = (2; 2; 2)

$\rho = \sqrt{x^2 + y^2} = \sqrt{(2)^2 + 2^2} = \sqrt{8} = \mathbf{2,83}$

$\varphi = arc\ tg\ \frac{y}{x} = arc\ tg\ \frac{2}{2} = arc\ tg\ (1) = \mathbf{45º}$

Z = 2

⇨ **A (2,83 ; 45º ; 2)**

b) B = (3; 2; 1)

$\rho = \sqrt{x^2 + y^2} = \sqrt{(3)^2 + 2^2} = \sqrt{13} = \mathbf{3,61}$

$\varphi = arc\ tg\ \frac{y}{x} = arc\ tg\ \frac{2}{3} = arc\ tg\ (0{,}666) = \mathbf{33º\ 41'\ 29,24''}$

Z = 1

⇨ **A (3,61 ; $\mathbf{33º\ 41'\ 29,24''}$; 1)**

9) Dadas las coordenadas cartesianas de los siguientes puntos hallar las coordenadas esféricas:

a) A = (3; 1; 3) **b) B = (-2; 2; -2)**

a) A = (3; 1; 3)

$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(3)^2 + 1^2 + 3^2} = \sqrt{19}$ = **4,36**

$\varphi = arc\ tg\ \frac{y}{x} = arc\ tg\ \frac{1}{3} = arc\ tg\ (0{,}333) = \mathbf{18º\ 26'\ 5,82''}$

$$\beta = arc\ seno\ \frac{z}{\rho} = arc\ seno\ \frac{3}{\sqrt{19}} = \mathbf{43º\ 28´\ 40"}$$

⇨ **A (4,36 ; 18º 26´ 5,82" ; 43º 28´ 40")**

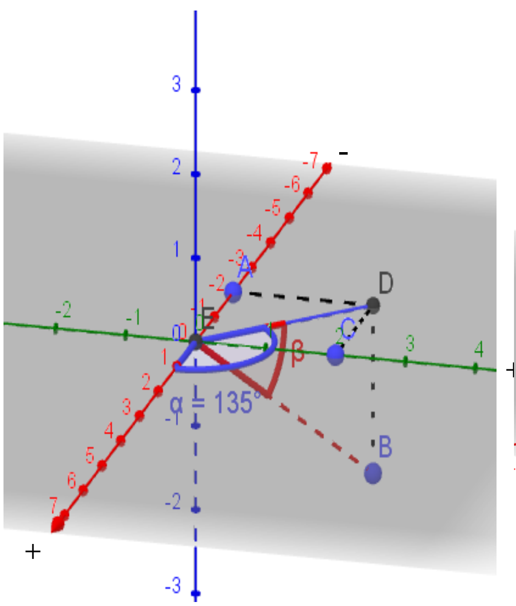
b) B = (-2; 2; -2)

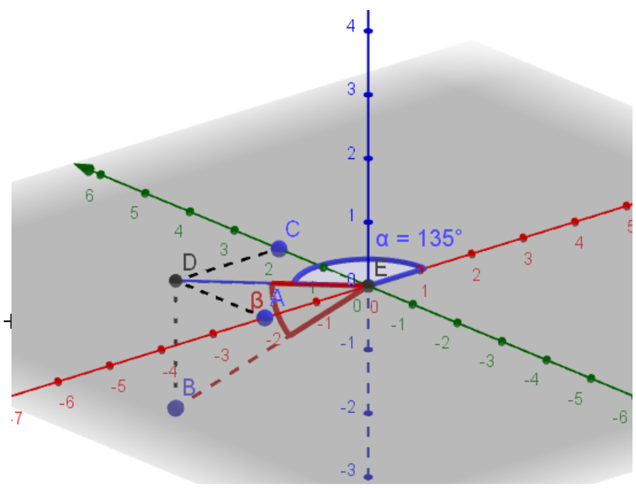
$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-2)^2 + 2^2 + (-2)^2} = \sqrt{12} = \mathbf{3{,}46}$$

$$\varphi = arc\ tg\ \frac{y}{x} = arc\ tg\ \frac{2}{(-2)} = -45º$$ **(le agrego 90º => 135º)**

$$\beta = arc\ seno\ \frac{z}{\rho} = arc\ seno\ \frac{(-2)}{\sqrt{12}} = \mathbf{-35º\ 18´\ 45"}$$

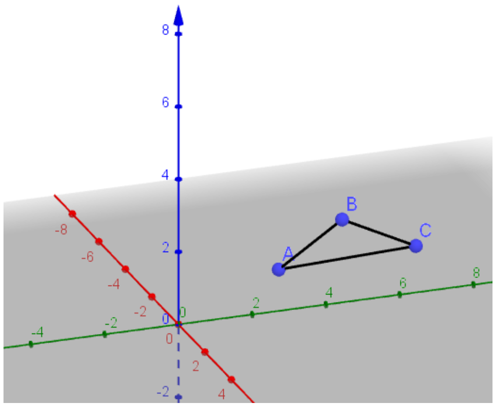
⇨ **A (3,46 ; 135º ; −35º 18´ 45"**





10) Dada un triángulo definido por sus vértices A, B y C, calcular las coordenadas polares de los vértices.

A = (2; 2; 2) B = (4; 3; 4) C = (4; 5; 3)



Vértice A)

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(2)^2 + 2^2 + 2^2} = \sqrt{12} = \mathbf{3{,}46}$$

$$\propto = \operatorname{arc\,cos} \frac{x}{\rho} = \operatorname{arc\,cos} \frac{(2)}{\sqrt{12}} = \mathbf{54º\ 41´\ 15{,}25''}$$

$$\beta = \operatorname{arc\,cos} \frac{y}{\rho} = \operatorname{arc\,cos} \frac{(2)}{\sqrt{12}} = \mathbf{54º\ 41´\ 15{,}25''}$$

$$\gamma = \operatorname{arc\,cos} \frac{z}{\rho} = \operatorname{arc\,cos} \frac{(2)}{\sqrt{12}} = \mathbf{54º\ 41´\ 15{,}25''}$$

⇨ **A (3,46 ; 54º 41´ 15,25 ; 54º 41´ 15,25"; 54º 41´ 15,25"**)

Vértice B)

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(4)^2 + 3^2 + 4^2} = \sqrt{41} = \mathbf{3{,}46}$$

$$\propto = \operatorname{arc\,cos} \frac{x}{\rho} = \operatorname{arc\,cos} \frac{(4)}{\sqrt{41}} = \mathbf{51º\ 19´\ 4''}$$

$$\beta = \operatorname{arc\,cos} \frac{y}{\rho} = \operatorname{arc\,cos} \frac{(3)}{\sqrt{41}} = \mathbf{62º\ 2´\ 49''}$$

$$\gamma = \operatorname{arc\,cos} \frac{z}{\rho} = \operatorname{arc\,cos} \frac{(4)}{\sqrt{41}} = \mathbf{51º\ 19´\ 4''}$$

⇨ **B (3,46 ; 51º 19´ 4" ; 62º 2´ 49" ; 51º 19´ 4"**)

Vértice C)

$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{(4)^2 + 5^2 + 3^2} = \sqrt{50}$ = **7,07**

$\propto = \text{arc cos } \frac{x}{\rho} = \text{arc cos } \frac{(4)}{\sqrt{50}} = \mathbf{55º\ 32´\ 39"}$

$\beta = \text{arc cos } \frac{y}{\rho} = \text{arc cos } \frac{(5)}{\sqrt{50}} = \mathbf{44º\ 59´\ 29"}$

$\gamma = \text{arc cos } \frac{z}{\rho} = \text{arc cos } \frac{(3)}{\sqrt{50}} = \mathbf{64º\ 53´\ 31"}$

⇨ **B (3,46 ;55º 32´ 39"; 44º 59´ 29" ; 64º 53´ 31")**

11) Dado el punto P = (3; 120º; 45º; 33º) (coord. polares) calcular sus coordenadas cartesianas.

$\cos \propto = \frac{x}{\rho} \Rightarrow x = \rho \, x \cos \propto = 3 \, x \cos 120º = \mathbf{-1,5}$

$\cos \beta = \frac{y}{\rho} \Rightarrow y = \rho \, x \cos \beta = 3 \, x \cos 45º = \mathbf{2,12}$

$\cos \gamma = \frac{z}{\rho} \Rightarrow z = \rho \, x \cos \gamma = 3 \, x \cos 33º = \mathbf{2,52}$

⇨ **P (-1,5 ; 2,12 ; 2,52)**

12) Dado el punto P = (4; 73º; 3) (coord. cilíndricas) calcular sus coordenadas cartesianas.

$\cos \varphi = \frac{x}{\rho} \Rightarrow x = \rho \, x \cos \varphi = 4 \, x \cos 73º = \mathbf{1,17}$

$\text{seno } \varphi = \frac{y}{\rho} \Rightarrow y = \rho \, x \text{ seno } \varphi = 4 \, x \text{ seno } 73º = \mathbf{3,82}$

Z = 3

⇨ **P (1,17 ; 3,82 ; 3)**

13) Dado el punto P = (4; 48º; 35º) (coord. Esféricas) calcular sus coordenadas cartesianas.

$\cos\varphi = \frac{x}{\rho} => x = \rho \; x \cos\beta \; x \cos\varphi \; = 4 \; x \cos 35º \; x \cos 48º = \mathbf{2,19}$

$\text{seno}\,\varphi = \frac{y}{\rho} => y = \rho \; x \cos\beta \; x \; \text{seno}\,\varphi \; = 4 \; x \cos 35º \; x \; sen \; 48º = \mathbf{2,43}$

$\text{seno}\,\beta = \frac{z}{\rho} => z = \rho \; x \; \text{seno}\,\beta \; = 4 \; x \; \text{seno} \; 35º = \mathbf{2,29}$

⇨ **P (2,19 ; 2,43 ; 2,29**)